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General Certificate of Education (A-level)
June 2013

Mathematics MPC2

(Specification 6360)

Pure Core 2

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

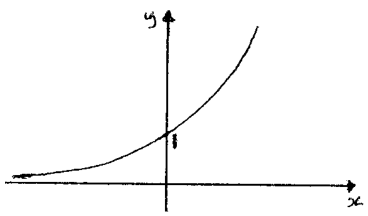
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

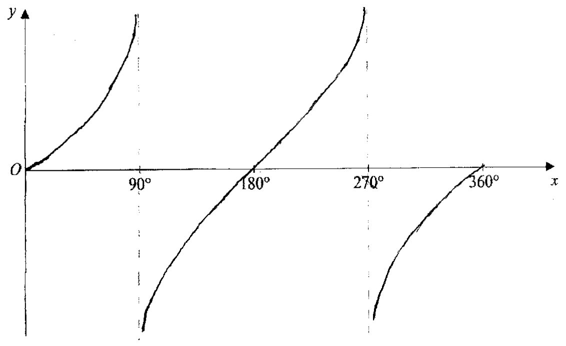
Q	Solution	Marks	Total	Comments
1(a)	20	B1	1	20
(b)	$\{S_\infty =\}\ \frac{a}{1-r} = \frac{80}{1-\frac{1}{2}}$	M1		$\frac{a}{1-r}$ <u>used</u> with $a = 80$ and $r = 0.5$ OE
	$\{S_\infty =\}\ 160$	A1	2	NMS 160 gets 2 marks unless rounding seen
(c)	$\{S_{12} =\}\ \frac{80\left(1-r^{12}\right)}{1-r} = 160(1-0.5^{12})$	M1		$\frac{80\left(1-r^{12}\right)}{1-r}$ seen (or used with r=0.5 OE)
	= 159.96(0937.) = 159.96 to 2dp	A1	2	Condone > 2dp
	Total		**5**	
2(a)	$\{\text{Arc} =\}\ r\theta = 20 \times 0.8$ …. = 16 (cm)	M1 A1	 2	$r\theta$ seen in **(a)** or used for the arc length
(b)	$\{\text{Area of sector} =\}\ \frac{1}{2}r^2\theta = \frac{1}{2}\times 20^2 \times 0.8$	M1		$\frac{1}{2}r^2\theta$ OE seen in **(b)** or used for the area
	…… = 160 (cm^2)	A1	2	
(c)	{Let D = angle ODB} $\frac{20}{\sin D} = \frac{15}{\sin 0.8}$	M1		Sine rule, ACF with sin D being the only unknown PI by next line
	$\sin D = \frac{20\times\sin 0.8}{15}$ $\left\{=\frac{14.3(471...)}{15}\right\}$ $\left\{=\frac{20}{20.9(10...)}\right\}$ = 0.956(474…) Acute 'D' = 1.27(467…)	m1		Correct rearrangement to 'sin$D$ = …' or to 'D =$\sin^{-1}$ (…)' OE. PI by at least 3sf correct value 1.27(467…) radians or 73(.033)º for acute angle or PI by at least 3sf value 1.86(692…) rounded or truncated for D .
	$D = \pi -$ Acute 'D' in rads	m1		<u>Dep on previous 2 marks</u> being awarded. PI by correct ft evaluation of $\pi -$c's acute D to at least 3 sf value or seeing 1.86(692…), rounded or truncated, for D
	{Angle ODB } = 1.87 {to 3sf}	A1	4	Condone >3sf.
	Total		**8**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$\{(2+y)^3=\}\ \ 8+12y+6y^2+y^3$	M1		At least 3 terms simplified and correct
		A1	2	All correct
(ii)	$(2+x^{-2})^3=8+12x^{-2}+6(x^{-2})^2+(x^{-2})^3$	M1		A replacement of y by x^{-2} in c's (a)(i) working. PI
	$(2-x^{-2})^3=8-12x^{-2}+6(x^{-2})^2-(x^{-2})^3$	A1F		Ft one incorrect coefficient in (a)(i) expansion.
	$(2+x^{-2})^3+(2-x^{-2})^3=16+12x^{-4}$	A1	3	CSO Be convinced. **SC2** for a fully correct solution, not using 'Hence'
(b)(i)	$\int\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\,dx=16x-4x^{-3}(+c)$	M1		Valid method to obtain the correct power of x after integrating qx^{-4}.
		A1F	2	$16x-4x^{-3}$ or $16x-4/x^3$ condone missing '+c'. Ft on c's p and q values. Coefficients and signs must be simplified
(ii)	$\int_1^2$......... $dx=[16(2)-4(2^{-3})]-[16-4]$	M1		F(2)−F(1) following integration **(b)(i)**
	$=31.5-12=19.5$	A1F	2	OE Ft on c's **<u>positive</u> <u>integer</u>** values of p and q. Since 'Hence' NMS scores 0/2
	Total		**9**	
4(a)		B1		Correct graph, must clearly go below the intersection pt and an indication of correct behaviour of curve for large positive and large negative values of x. Ignore any scaling on axes.
		B1	2	Only one y-intercept, marked/stated as 1 or as coords (0, 1) with graph having no other intercepts on either axes.
(b)	$9^x=15 \Rightarrow x\log 9=\log 15$	M1		OE eg $x=\log_9 15$
	$(x=)\ 1.23(2486\ldots)=1.23$ to 3sf	A1	2	Condone > 3sf. Must see evidence of logs used so NMS scores 0/2
(c)	$\{f(x)=\}\ \ 9^{-x}$	B1	1	OE
	Total		**5**	

Q	Solution	Marks	Total	Comments
5(a)	$h = 0.5$ $f(x) = \sqrt{8x^3+1}$ $I \approx \frac{h}{2}\{f(0)+f(2)+2[f(0.5)+f(1)+f(1.5)]\}$	B1 M1		$h = 0.5$ stated or used. $I \approx \frac{h}{2}\{f(0)+f(2)+2[f(0.5)+f(1)+f(1.5)]\}$ OE
	$\frac{h}{2}$ with $\{\ldots\} = \sqrt{1}+\sqrt{65}+2\left(\sqrt{2}+\sqrt{9}+\sqrt{28}\right)$ $= 1 + 8.06... + 2(1.41\ldots + 3 + 5.29\ldots)$ $= 9.0622\ldots + 2\times 9.7057\ldots$	A1		OE Accept 1dp evidence. Can be implied by later correct work provided more than one term or a single term which rounds to 7.12
	$(I \approx)\ 0.25[28.47\ldots]\ \{= 7.118..\} = 7.12$ (to 3sf)	A1	4	CAO Must be 7.12
(b)	Stretch(**I**) in x-direction(**II**) scale factor 2 (**III**)	M1		Need (**I**) **and** either (**II**) or (**III**)
		A1	2	Need (**I**) and (**II**) and (**III**) More than 1 transformation scores 0/2
(c)	$g(x) = \sqrt{(x-2)^3+1} - 0.7$	M1		$\sqrt{(x-2)^3+1}-0.7$ or $\sqrt{(x-2)^3+1}+0.7$ or $\sqrt{(x+2)^3+1}-0.7$ or $\sqrt{(x-2)^3+1-0.7}$ or their equivalents
		A1		$\sqrt{(x-2)^3+1}-0.7$ OE
	$g(4) = 2.3$	A1	3	2.3 OE
	<u>**Altn**</u>			
	(4, …) on $y = g(x)$ comes from translating (2, 3) on $y=\sqrt{x^3+1}$	(M1) (A1)		from (2, ….) on $y=\sqrt{x^3+1}$ from (2, 3) on $y=\sqrt{x^3+1}$
	(2, 3) after translation becomes (4, 2.3) so $g(4) = 2.3$	(A1)	(3)	2.3 OE
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)	$\sqrt{x} = x^{0.5}$	B1		$\sqrt{x} = x^{0.5}$ or $\sqrt{x} = x^{\frac{1}{2}}$ seen or used
	$\dfrac{12 + x^2\sqrt{x}}{x} = \dfrac{12 + x^{2.5}}{x}$ $= 12x^{-1} + x^{1.5}$	B1		$12x^{-1}$ or $p = -1$
		B1	3	$x^{1.5}$ or $q = \dfrac{3}{2}$ (=1.5)
(b)(i)	$\dfrac{dy}{dx} = -12x^{-2}$	B1F		Ft on c's p only if c's p is a negative integer
	$+1.5x^{0.5}$	B1F	2	Ft on c's q only if c's q is a pos non-integer
(ii)	When $x = 4$, $y = 11$	B1		
	When $x = 4$, $\dfrac{dy}{dx} = \dfrac{-12}{16} + 3 = \dfrac{9}{4}$	M1		Attempt to find $\dfrac{dy}{dx}$ when $x = 4$ PI
	Gradient of normal = $-\dfrac{4}{9}$	m1		$m \times m' = -1$ used
	Eqn of normal: $y - 11 = -\dfrac{4}{9}(x - 4)$	A1	4	ACF eg $4x + 9y = 115$
(iii)	At St Pt $\dfrac{dy}{dx} = -12x^{-2} + 1.5x^{0.5} = 0$	M1		Equating c's $\dfrac{dy}{dx}$ to zero.
	$\Rightarrow x^2x^{0.5} = 8$, $\Rightarrow x^{\frac{5}{2}} = 8 \Rightarrow x = 8^{\frac{2}{5}}$	A1		A correct eqn in the form $x^n=c$ or $x = c^{\frac{1}{n}}$ correctly obtained.
	$\Rightarrow x = (2^3)^{\frac{2}{5}} \Rightarrow x = 2^{\frac{6}{5}}$	A1	3	CSO $x = 2^{\frac{6}{5}}$. All working must be correct and in an exact form. If 'x=0' also appears then A0 CSO
			12	

Q	Solution	Marks	Total	Comments
7(a)	$72 = 96p + q$ $24 = 24p + q$	M1 M1		 OE
	$48 = 72p$	m1		Valid method to solve the correct two simultaneous eqns in p and q to at least the stage $48 = 72p$ OE
	$p \left(= \frac{48}{72}\right) = \frac{2}{3}$	A1	4	AG CSO
(b)	$q = 8$	B1		Award if seen at any stage in Q7
	$u_3 = 48 + q \qquad (u_3 =)\ 56$	B1F	2	If not 56, ft on (48 + c's q) provided at least M1 scored in part (a).
	Total		**6**	
8(a)	$b = a^c$	B1	1	
(b)	$2\log_2(x+7) - \log_2(x+5) = 3$ $\log_2(x+7)^2 - \log_2(x+5) = 3$	M1		A law of logs used correctly on a correct expression.
	$\log_2 \frac{(x+7)^2}{x+5} = 3$	M1		A further correct use of law of logs on a correct expression.
	$= 3\log_2 2 = \log_2 2^3$ $\Rightarrow \frac{(x+7)^2}{x+5} = 2^3$	B1		$3 = 3\log_2 2$ or $3 = \log_2 2^3$ (= $\log_2 8$) seen or eg $\log f(x) = 3 \Rightarrow f(x) = 2^3$ (=8) OE
	$\Rightarrow (x+7)^2 = 8(x+5)$	A1		Correct equation having eliminated logs and fractions
	$\Rightarrow x^2 + 14x + 49 = 8x + 40$ $\Rightarrow x^2 + 6x + 9\ (= 0)$	A1		
	Since $6^2 - 4(1)(9) = 0$, (there is only) one value of x (which satisfies the given equation).	A1	6	OE CSO Need conclusion which is also correctly justified
	Total		**7**	

Q	Solution	Marks	Total	Comments
9(a)(i)		B1 B1 B1	3	Ignore any part of the graph drawn outside interval $0° \le x \le 360°$ in (a) A 3 branch curve between 0 and 360 meeting the x-axis at or very close to 0, 180, 360 only A 3 branch curve between 0 and 360 with correct shape tending to infinity at, at least 3, of the 4 relevant ends Correct graph for $0° \le x \le 360°$, with correct intercepts. Asymptotes not explicitly required but graphs should show correct 'tendency' close to 90 and 270.
(ii)	135° ; 315°	B2,1,0	2	B2 for both 135 and 315 and no 'extras' in interval $0° \le x \le 360°$ (If not B2 then award B1 for either 135 or 315 with or without extras)
(b)(i)	$6\tan\theta\sin\theta = 5 \Rightarrow 6\dfrac{\sin\theta}{\cos\theta}\sin\theta = 5$	M1		$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ used
	$6\dfrac{\sin^2\theta}{\cos\theta} = 5 \Rightarrow 6\dfrac{1-\cos^2\theta}{\cos\theta} = 5$	m1		$\sin^2\theta$ replaced by $1-\cos^2\theta$ throughout
	$6 - 6\cos^2\theta = 5\cos\theta \Rightarrow 6\cos^2\theta + 5\cos\theta - 6 = 0$	A1	3	Completion AG Be convinced
(ii)	$6\tan 3x\sin 3x = 5 \Rightarrow 6\cos^2 3x + 5\cos 3x - 6 = 0$	M1		Using (b)(i) with θ=3x PI by attempting to solve eg for theta then dividing soln(s) by 3
	$(3\cos 3x - 2)(2\cos 3x + 3)$ (=0) (cos3x = 2/3, −3/2)	m1		Correct factorisation or correct subst into the quadratic formula PI by two 'correct' roots
	$\cos 3x = \dfrac{2}{3}$ = cos 48.1(89..) [= cos α] 3x = α , 360−α , 360+α .	m1		Dep on M1 only, 3x =α , 360−α , 360+α for c's α. from an eqn cos3x = k where −1<k<1 OE PI and no solns from k outside $-1 \le k \le 1$
	x = 16º, 104º, 136º	B1 B1 B1	6	AWRT 16, 104, 136. Deduct one mark (from any award of these 3 B marks) if more than three solns given inside the interval $0° \le x \le 180°$. Ignore any solutions outside the interval $0° \le x \le 180°$. NMS Max. is B3/6
	Total		**14**	
	TOTAL		**75**	